

Antiferromagnetism Molecular field for $J < 0$: two sublattices $\uparrow M_1, \downarrow M_2$

$$M_1 = \chi_0(aM_2 + H)$$

$$M_2 = \chi_0(aM_1 + H)$$

$$a = \frac{\sum_j 4zJ_j}{N(g\mu_B)^2} < 0 \quad \chi_0 = \frac{C}{2T}$$

add two equations

$$M = M_1 + M_2 = \chi_0(a(M_1 + M_2) + 2H) = \chi_0(aM + 2H)$$

$$\chi = \frac{M}{H} = \frac{2\chi_0}{1 - \chi_0 a} = \frac{C}{T - \theta}$$

$M_1, M_2 \rightarrow$ matrix

$$\begin{pmatrix} -\frac{1}{\chi_0} & a \\ a & -\frac{1}{\chi_0} \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} H \\ H \end{pmatrix}$$

Determinant = 0 at $H \rightarrow 0$

$$\begin{vmatrix} -2T/C & a \\ a & -2T/C \end{vmatrix} = 0 \rightarrow T_N = |Ca/2| = |\theta|$$

$\chi \perp$ hard axis spin \perp
 $\chi //$ easy axis // Spin
 Néel $T \sim |Weiss T|$
 Transition Interaction temperature < 0
 Paramagnetism

Easy axis of antiferromagnets

Spins in antiferromagnet are oriented in a particular direction due to

- 1) Dipole interaction
- 2) crystal field

Increasing H along the easy axis leads to abrupt change to spin $\perp H$ at H_{sf} .

$$H_{sf} = \sqrt{2Ka}$$

easy axis: spin $// H$
 Hard axis: spin $\perp H$
 spin flop
 a : Molecular field coefficient
 K : magnetic anisotropy (difference of $//$ and \perp)

With increasing $H //$ easy axis, suddenly spin becomes $\perp H$ at H_{sf}

Energy difference between easy and hard axis K (magnetic anisotropy) **spin flop**

$$\frac{1}{2} \chi_{\perp} H^2 - \frac{1}{2} \chi_{//} H^2 = K \rightarrow H_{sf} = \sqrt{\frac{2K}{\chi_{\perp} - \chi_{//}}} = \sqrt{2K|a|}$$

Ferrimagnetism

Alternating spins with different S (e.g. different metals) lead to remaining moment even for antiparallel order for $J < 0$.

Ferrite Fe_3O_4 has Fe^{3+} and Fe^{2+} .
Most molecular magnets.

Magnetic frustration

Including a' from the next neighbor

$$M_1 = \chi_0(aM_2 - a'M_1 + H)$$

$$M_2 = \chi_0(aM_1 - a'M_2 + H)$$

Add two equations

$$M = M_1 + M_2 = \chi_0((a - a')(M_1 + M_2) + 2H) = \chi_0((a - a')M + 2H)$$

or

$$\theta = \frac{2(zJ - z'J')}{k_B}$$

loss of a' (frustration)

Antiferromagnetism does not happen in a triangular lattice because the third spin orientation is not determined (spin liquid).

Low dimensional fluctuation

1D Magnet does not exist (Landau, Lifshitz Statistical Physics)

Statistical weight of a state with n boundaries in a 1D ferromagnet (length L):

$$W = \frac{L!}{(L-n)!n!}$$

$$\ln W = L \ln L - (L-n) \ln(L-n) - n \ln n$$

$$= L \ln \frac{L}{L-n} + n \ln \frac{L-n}{n} \approx -n \ln \frac{n}{L} \quad n \ll L \rightarrow \text{第1項} \sim 0$$

The free energy is $G = G_0 - TS + nJ = G_0 + k_B T n \ln \frac{n}{L} + nJ$

Realized n is obtained by minimizing G :

$$\frac{\partial G}{\partial n} = k_B T \ln \frac{n}{L} + J = 0$$

$\ln(n/L)$ takes a large negative value for small n/L , so near $n \sim 0$: $\frac{\partial G}{\partial n} < 0$

With increasing n , G decreases.

Accordingly, there is a minimum of G at finite $n \neq 0$.

For finite $T \neq 0$, many boundaries appear, and long-range magnetic order is never established. Then there is no 1D ferromagnet.

Multi dimension increases the loss of J , and leads to long range magnetic order.

In general, a 1D system does not undergo any phase transitions at finite T .

Low-dimensional magnet : large J only for 1 or 2 directions

Curie low-dimensional fluctuation

This peak reflects the short-range order due to J , but a real phase transition does not take place.

Approaches to the Curie law at high T .

$\chi(T)$ is fitted to numerical calculations Bonner & Fischer, *Phys. Rev. A* **135**, 640 (1964). de Jongh & Miedema, *Experiments on Simple Magnetic Model Systems*, Taylor (1974).

3D long-range order due to interchain interaction. (反強磁性)

β' -(BEDT-TTF)₂ICl₂

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2次元正方格子

1次元交互鎖 ($\alpha=0.5$)

1次元Heisenberg

Curie

Fitting to polynomials

1次元Heisenberg, W. E. Estes et al. *Inorg. Chem.* **17**, 1415 (1978).
 1次元交互Heisenberg, J. M. Hall et al. *Inorg. Chem.* **20**, 1033 (1981).
 2次元正方格子, M. E. Lines, *J. Phys. Chem. Solids*, **31**, 101 (1970);
 J. Wang, *Phys. Rev. B* **44**, 2396 (1991).
 2次元三角格子, N. Elstner, et al. *Phys. Rev. Lett.* **71**, 1629 (1993);
 M. Tamura and R. Kato, *J. Phys. Cond. Matter.* **14**, L729 (2002).
 1次元フェリ, R. Georges et al. *Magnetism: Molecules to Materials*, ed. by J. S. Miller and M. Drillon, Wiley (2001), p1.

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Electron correlation

Coulomb repulsion in molecular orbital theory

Configuration interaction

Electron correlation in solids

- Hubbard model
- Stoner model
- Mott insulator
- Strict solution of Hubbard model

Origin of magnetic interaction J from the Hubbard model

Charge order : non half-filled case

Hartree-Fock approximation of the Hubbard model

Superexchange interaction

Charge density waves

Kondo effect

Localization in a disordered system

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Electron Correlation (電子相関)

Molecular orbital theory considering inter-electron Coulomb repulsion

$$H = \sum_i \left[-\frac{\hbar^2}{2m} \nabla_i^2 - \frac{1}{4\pi\epsilon_0} \sum_n \frac{Z_n e^2}{r_{ni}} \right] + \frac{1}{4\pi\epsilon_0} \sum_{i \neq j} \frac{e^2}{r_{ij}}$$

Sum for electrons Attraction from nuclei Inter-electron Coulomb repulsion

For an N -electron system, in general

$$\Psi(1,2,\dots) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \varphi_1(1) & \varphi_1(2) & \dots & \varphi_1(N) \\ \varphi_2(1) & \varphi_2(2) & \dots & \varphi_2(N) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_N(1) & \varphi_N(2) & \dots & \varphi_N(N) \end{vmatrix} \quad \text{Slater determinant}$$

$$= \frac{1}{\sqrt{N!}} \sum_P P^{(-1)} \varphi_1(1)\varphi_2(2)\dots\varphi_N(N)$$

All permutations for electron numbers
 One permutation such as $1 \leftrightarrow 2$ generates a minus sign.

Exchange $1 \leftrightarrow 2$ electrons \rightarrow Columns 1,2 exchange \rightarrow (Antisymmetric)
 The same φ_1 and $\varphi_2 \rightarrow$ The same 1, 2 lines \rightarrow |Determinant|=0
 \rightarrow 2 electrons (Fermi粒子) cannot take the same state (Pauli's exclusion principle)

Put the Slater determinant in $E = \int \Psi^* H \Psi d\tau$ to give

$$E = \sum_k H_k + \sum_{k \neq l} (J_{kl} - K_{kl})$$

One-electron part K : # of MO

$$H_k = \int \varphi_k^* \left[-\frac{\hbar^2}{2m} \nabla^2 - \frac{1}{4\pi\epsilon_0} \sum_n \frac{Z_n e^2}{r_n} \right] \varphi_k d\tau$$

Coulomb integral

$$J_{kl} = \int \varphi_k^*(1) \varphi_l^*(2) \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_{12}} \right) \varphi_k(1) \varphi_l(2) d\tau$$

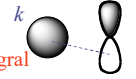
Average el-el repulsion between MO k with electron distribution ρ_k and MO l

$$= \int \frac{e^2 \rho_k(1) \rho_l(2)}{4\pi\epsilon_0 r_{12}} d\tau > 0$$

Exchange integral

$$K_{kl} = \int \varphi_k^*(1) \varphi_l^*(2) \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_{12}} \right) \varphi_k(2) \varphi_l(1) d\tau$$

Exchange 1 and 2




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(1) $\int \alpha(1)\beta(1)d\tau = 0$ so that for opposite spins: $K_{kl} = 0$

The same spin cannot come to the same place due to the Pauli exclusion principle (Fermi's principle). This reduces the Coulomb repulsion. The correction for J_{kl}

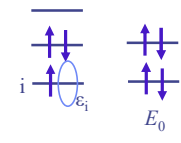
(2) 2 electrons in a MO $E = 2H_1 + J_{11}$



(3) Remove one electron from the i-th MO.


$$E = E_0 - \left[H_i + \sum_k (J_{ik} - K_{ik}) \right]$$

Effective energy of the i-th electron (Koopmans' Theorem)



Total energy is $E_0 = \sum_i \epsilon_i$: a sum of the one-electron energies

One electron is added on the j-th MO

$$E = E_0 + [H_j + \sum_k (J_{jk} - K_{jk})] = E_0 + \epsilon_j$$


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(5) Stability of 2-level 2-electron states

(a) $E_1 = J_{11}$ (singlet state)

(b) $E_2 = \Delta\epsilon + J_{12} - K_{12}$ (Triplet state, 磷光 Phosphorescence)

(c) $E_3 = \Delta\epsilon + J_{12}$ (Singlet state, 荧光 Fluorescence)

$K_{12} > 0 \rightarrow (c) > (b)$

Triplet excitation is stabler than singlet excitation.

(a) (b) from $H_1 + J_{11} < H_2 + J_{12} - K_{12}$

1, 2 are MOs on the same molecule \rightarrow assuming $J_{11} \sim J_{12}$

$H_2 - H_1 > K_{12} \rightarrow (b) > (a) \rightarrow$ Chemical bond leads to singlet

$H_2 = H_1$ Degenerate (a) > (b) \rightarrow Triplet **Hund's rule**

Degeneracy leads to triplet: triplet instability

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Molecular Correlation in Molecular Orbital

$\alpha - \beta$ $\phi = \chi_A - \chi_B$

A hydrogen atom has 2 electrons in the bonding orbitals, so that the whole wave function is

$\alpha + \beta$ $\phi = \chi_A + \chi_B$

$$\Psi = (\chi_A(1) + \chi_B(1))(\chi_A(2) + \chi_B(2))(\alpha(1)\beta(2) - \alpha(2)\beta(1))$$

Spin part in the Slater determinant is antisym. \rightarrow singlet \rightarrow omitted

$$\rightarrow \chi_A(1)\chi_A(2) + \chi_A(1)\chi_B(2) + \chi_B(1)\chi_A(2) + \chi_B(1)\chi_B(2)$$

2 electrons on hydrogen A: $\uparrow\downarrow$ (H-H⁺ ionic)

Covalent bond: $\uparrow\downarrow + \uparrow\downarrow$

2 electrons on hydrogen B: $\uparrow\downarrow$ (H-H⁺ ionic)

Ionic contribution in MO is 50%; this is obviously too large!

This comes from the averaged electron density, and electron correlation (電子相関) is not considered.

cf. Valence Bond Theory (Heitler Lindon法)

Use $\Psi = \chi_A(1)\chi_B(2) + \chi_B(1)\chi_A(2)$ instead of the above Ψ

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Configuration Interaction
 CI (配置間相互作用)

Linear combination of an excited state is included in order to involve electron correlation in MO:

$$\Psi = \Psi_1 + c\Psi_2$$

$$= (\chi_A(1) + \chi_B(1))(\chi_A(2) + \chi_B(2)) + c(\chi_A(1) - \chi_B(1))(\chi_A(2) - \chi_B(2))$$

$$= (1+c)(\chi_A(1)\chi_A(2) + \chi_B(1)\chi_B(2)) + (1-c)(\chi_A(1)\chi_B(2) + \chi_B(1)\chi_A(2))$$

Ionic Covalent 共有結合性

$c = -1$ corresponds to the valence bond method, with no ionic contribution.
 $c = 0$ corresponds to the MO method with 50% ionic contribution.
 c is determined so as to minimize the energy ($-1 < c < 0$).

CI calculation is inevitable for the calculation of excited states in MO.

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Mathematical Models Representing Mott Insulator

(1) Gutzwiller 波動関数

$$\Psi_G = \prod_i (1 - \eta n_{i\uparrow} n_{i\downarrow}) \Psi_0 = \exp(-\eta \sum_i n_{i\uparrow} n_{i\downarrow}) \Psi_0$$

Subtract double occupancy (ionic state) from the wave function. Similar to CI, but the calculation is difficult.

(2) Hubbard Model

Double occupancy costs energy U (on-site Coulomb repulsion).

$$H = \sum_{i \neq j} t_{ij} a_i^\dagger a_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Cost U

Energy band $2t \cos ka$ Double occupancy at the i site costs energy U . Zero if either $n_{i\uparrow}$ or $n_{i\downarrow}$ is 0.

Secondary quantization
 a_j : annihilate i -site electron a_i^\dagger : create i -site electron
 $n_{i\uparrow} = a_i^\dagger a_i$: i -site electron number

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When we assume U does not change the band shape,

$$E_\uparrow = 2t \cos ka$$
 Energy band of the 1-st electron

$$E_\downarrow = 2t \cos ka + U$$
 Energy band of the 2-nd electron

(1) $t \ll U$

Split \uparrow and \downarrow energy bands by U .
 $U > W$ makes a Hubbard gap, and becomes insulating when half-filled.
 Mott transition happens when U exceeds W .

Lower Hubbard band occupies up to 1 electron (usually 2).
 Spin orientation may be random (paramag).
 Maybe antiferromagnetic at low T .

Hubbard model is insulating only for half-filled. (other fillings are metals)

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Fully Occupied		Insulator
Partially Occupied (less than half filled)		Metal
Half Filled		Mott Insulator (U ≠ 0)
Partially Occupied (more than half filled)		Metal

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Mean-field solutn. of Hubbard model (Stoner model)

In the ferromagnetic metal with different \uparrow and \downarrow electron numbers, the magnetization is

$$m = n_{\uparrow} - n_{\downarrow} = \int_{E_F - \Delta}^{E_F + \Delta} D(E) dE = 2D\Delta$$

Kinetic energy increases by this process as:

$$\Delta E = \int_0^{E_F + \Delta} E D dE + \int_0^{E_F - \Delta} E D dE - 2 \int_0^{E_F} E D dE$$

$$= \int_{E_F - \Delta}^{E_F + \Delta} E D dE = D \Delta^2 = \frac{m^2}{4D} = \frac{1}{4} W m^2$$

From the above Assuming bandwidth $W \sim 1/D$

This spin polarization leads to energy gain of the U -term、 (using $n = n_{\uparrow} + n_{\downarrow}$, $m = n_{\uparrow} - n_{\downarrow}$) $E = U n_{\uparrow} n_{\downarrow} = \frac{U}{4} (n^2 - m^2)$

Add the above two $E = \frac{W-U}{4} m^2 + \frac{U}{4} n^2$ $n = \text{total el \#} = \text{const.}$

$W > U$ E min. is $m = 0$ 常磁性金属 paramagnetic metal
 $W < U$ E min. is $m \neq 0$ 磁性金属 magnetic metal

When U exceeds W (Stoner limit), electron spin polarizes to make a magnetized metal.

Applying weak magnetic field h

$$E = \frac{W-U}{4} m^2 + \frac{U}{4} n^2 - m h$$

$$\rightarrow \frac{1}{\chi} = \frac{\partial h}{\partial m} = \frac{W-U}{2}$$

or because the energy of the spin system is $\frac{1}{2\chi} m^2$

Susceptibility diverges to ∞ at $U \rightarrow W$ (the same in m^* and DOS $D(E_F)$)

Magnetic susceptibility

$$\chi = \frac{2}{W-U} = \frac{\chi_0}{1-UD_0}$$

The same as a ferromagnet approaching to T_c from $T > T_c$
 Here $T=0$, and the horizontal axis is U/W

RPA result:
 W. F. Brinkman and T. M. Rice, *Phys. Rev. B* 2, 4302 (1970).

Electron Correlation in Solids

Energy band with 1 electron - 1 site state

Mott insulator $\rho \propto \exp(E_g/k_B T)$

Magnet (Curie-Weiss則) $\chi = C/(T-\theta)$

Localized Small ionic contribution 原子価結合法的

Potential $E. >$ Kinetic Energy

電子密度小 Bandwidth小 電子雲overlap小 t 小

Pressure enhances t A transition from metal to insulator (Mott転移)

half-filled band

Metal $\rho \propto T$

No magnetism (Pauli常磁性) $\chi = \text{const.}$

Delocalized (itinerant) Large ionic contribution 分子軌道法的

Potential $E. <$ Kinetic Energy

電子密度大 Bandwidth大 t 大 電子雲overlap大

Metal-insulator transition in κ phase organic superconductors

κ -(BEDT-TTF)₂Cu[C(CN)₂]Cl
 κ -(BEDT-TTF)₂Cu[C(CN)₂]Br
 κ -(BEDT-TTF)₂Cu(NCS)₂

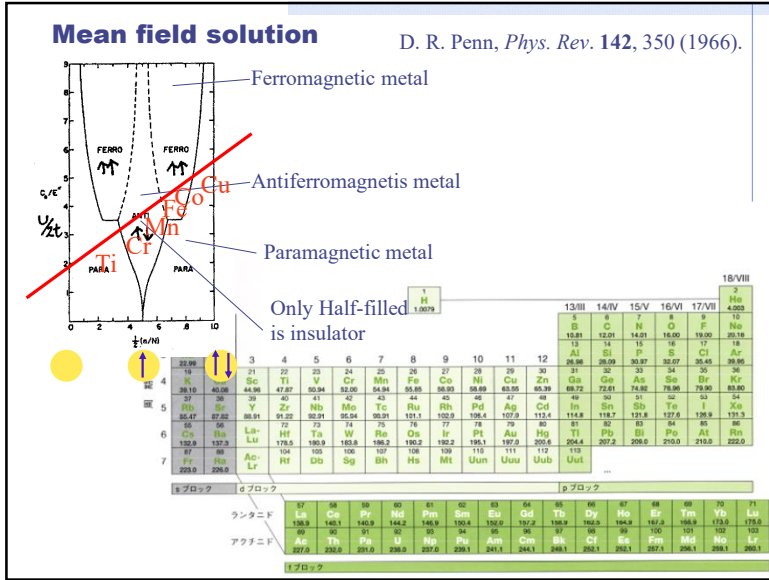
Thermal contraction shifts the boundary right
 Critical (end) point like Liquid-gas boundary

Temperature / K

Resistivity

Pressure

Universal phase diagram κ phase (Kanoda Diagram)
 K. Kanoda, *Hyperfine Interact.* 104, 235 (1997).



Strict solutions of the Hubbard model

Antiferromagnetic insulator

Strict solution only for one-dimensional case
 Only half-filled is insulator for infinitesimally small $U \neq 0$.
 E. H. Lieb and F. Y. Wu, *Phys. Rev. Lett.* **20**, 1445 (1968).
 川上、梁、共系場理論と一次元電子系 岩波

Numerical calculation for more than two dimensions
 J. E. Hirsch, *Phys. Rev. B* **31**, 4403 (1985).

upper Hubbard band
 Hubbard gap
 lower Hubbard band

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Derivation of magnetic interaction J from the Hubbard model

2 site 2 electron Hubbard model

Singlet solution $4C_2=6$ states Triplet solution

① $\uparrow\downarrow$	$-E$	t	0	t	$= 0$ $E = 0, 0$
② $\uparrow\uparrow$	t	$U-E$	t	0	
③ $\downarrow\downarrow$	0	t	$-E$	t	
④ $\uparrow\downarrow$	t	0	t	$U-E$	

By one electron jump ④ has by U higher energy
 ① changes to ④

The solution is

$$E = \frac{-E \pm \sqrt{U^2 + 16t^2}}{2}$$

The $E=0$ solution $\Psi = \frac{①-②}{\sqrt{2}}$ is asymmetric and goes to triplet.

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2 site 2 electron Hubbard model

上村 電子物性論 培風館(1995), p30

$E = \frac{U + \sqrt{U^2 + 16t^2}}{2}$
 $E = \frac{U - \sqrt{U^2 + 16t^2}}{2} \rightarrow -\frac{4t^2}{U} (U \gg t)$
 $U \gg t \rightarrow J = E_T - E_S = -\frac{4t^2}{U}$

Including the exchange integral K ,

$$J = K - \frac{4t^2}{U}$$

Potential exchange $J > 0$ Ferromagnetic
 Kinetic exchange $J < 0$ Antiferromagnetic

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$J = K - \frac{4t^2}{U}$

(1) Potential exchange
 In degenerated 2 levels, triplet is more stable by: J_{12} $J_{12} - K_{12}$
 $K_{12} = \int \varphi_1^*(1)\varphi_2^*(2) \frac{e^2}{r_{12}} \varphi_1(2)\varphi_2(1) d\tau > 0$ (exchange integral)
 → spin parallel (ferromagnetic) (=Hund's rule)

(2) Kinetic exchange
 Electron moves from atom 1 to atom 2 by transfer integral t .
 Energy increases by the on-site Coulomb integral U (b). When going back to (a) and mixing the (b) state, the (a) state is stabilized by $J = -2t^2/U$.
 Parallel spin (c) is not stabilized due to the Pauli exclusion principle, So antiparallel (a) is more stable by $J = -2t^2/U$.

Usually (2) >> (1) leads to antiferromagnetism, and $J < 0$.

Exception : Orthogonal nearby orbitals leads to $t \rightarrow 0$ so that (2) \rightarrow 0, (1) remains to result in ferromagnetism $J > 0$ (Kanamori-Goodenough rule).

Charge order in non half-filled bands
Only half filled is insulating in the Hubbard model
 → **Artifact of on-site U ?**

$H = \sum_{i \neq j} t_{ij} a_i^\dagger a_j + U \sum_i n_{i\uparrow} n_{i\downarrow} + V \sum_{i \neq j} n_i n_j$

extended Hubbard model

Adjacent-site electron leads to by V (off-site Coulomb repulsion) higher energy

At 1/4-filled, alternate arrangement is stable.

Charge order insulator

2-fold periodicity
 (= $4k_F$ CDW in a one-dimensional system)
 (cf. no new periodicity in a Mott insulator)

Extended Hubbard model including V

$H = \sum_{i \neq j} t_{ij} a_i^\dagger a_j + U \sum_i n_{i\uparrow} n_{i\downarrow} + V \sum_{i \neq j} n_i n_j$

V is approximately inverse of the distance between molecular centers

$U = J_{11}$ $V = J_{12}$

2 orbitals on the same position → U

Coulomb repulsion is $\propto 1/r$: not only on-site
Inverse distance in Bohr radius (0.52 Å) → V in Hartree (26.6 eV)

Charge order in 1D : atomic limit ($t \rightarrow 0$)

$H = \sum_{i \neq j} t_{ij} a_i^\dagger a_j + U \sum_i n_{i\uparrow} n_{i\downarrow} + V \sum_{i \neq j} n_i n_j$

$\langle n_i \rangle = \langle n_i \rangle = n/2$

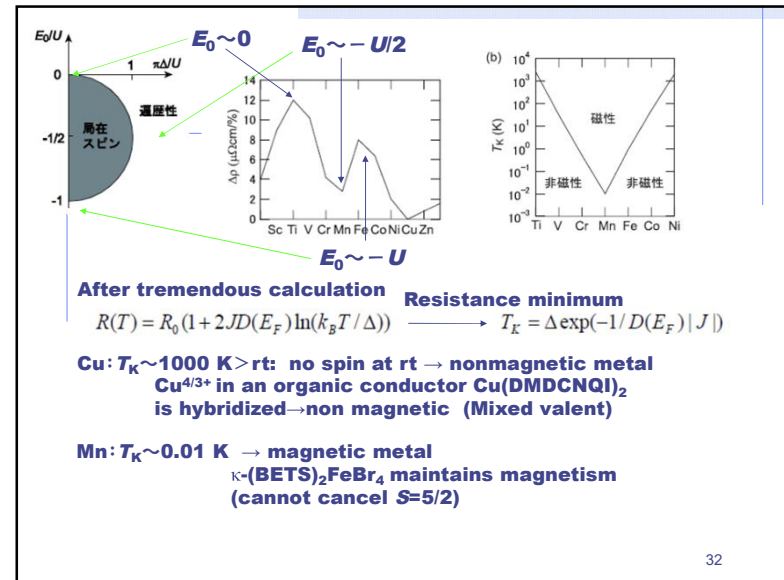
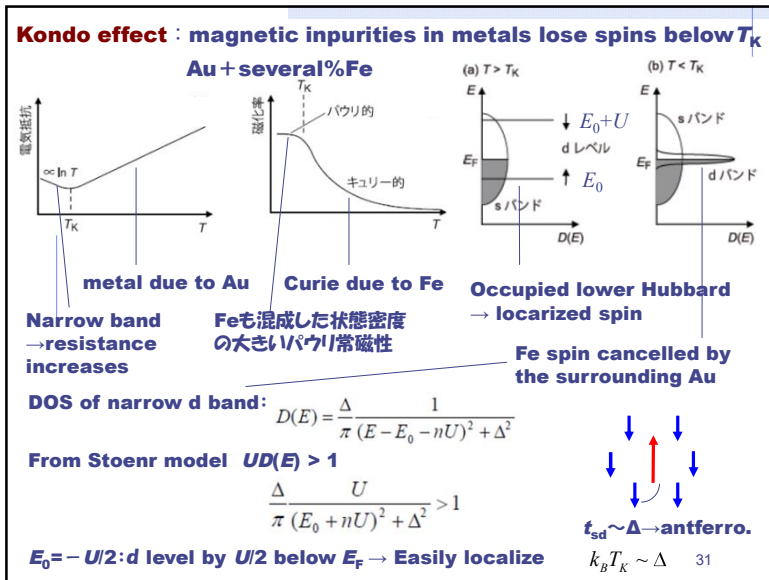
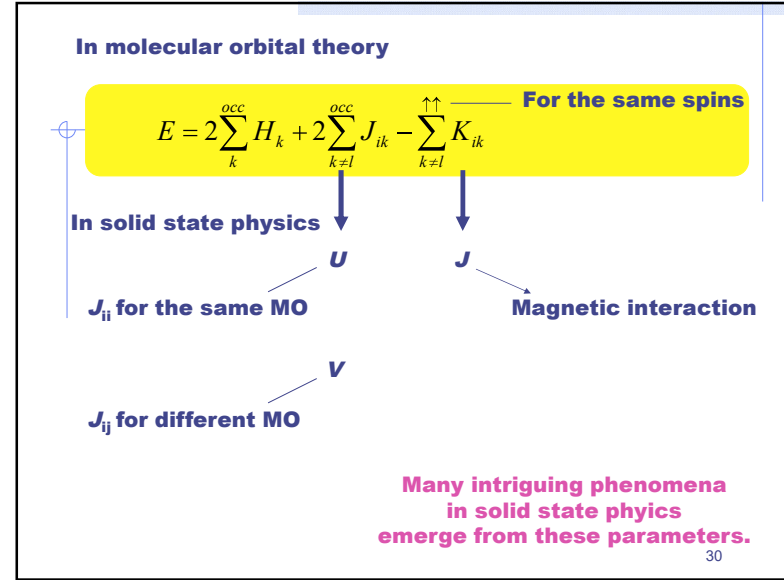
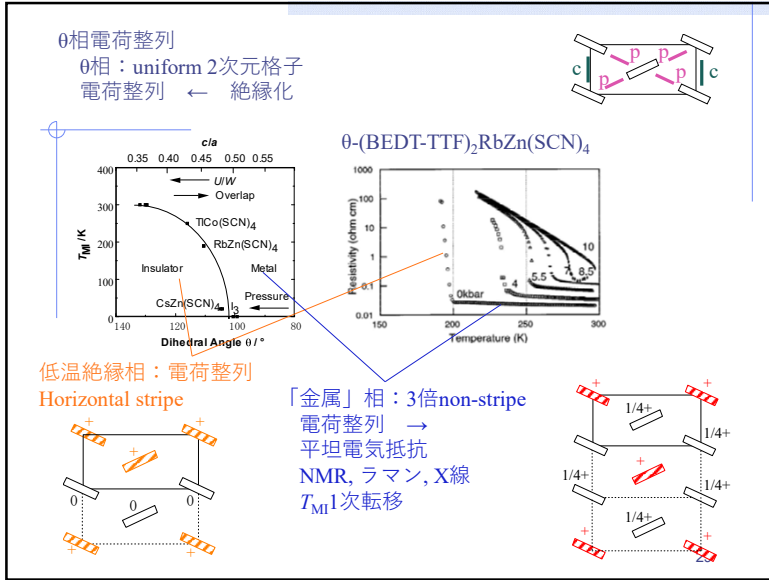
(a) 金属 $n = 1/2$ スピン分極なし $n = 1/2$
 $E = U \times \frac{1}{4} \times \frac{1}{4} + V \times \frac{1}{2} \times \frac{1}{2} = \frac{U}{16} + \frac{V}{4}$

(b) 電荷整列 $n = 1$ スピン分極なし
 $E = \frac{1}{2} [U \times (\frac{1}{2} \times \frac{1}{2} + 0 \times 0) + 2V \times 1 \times 0] = \frac{U}{8}$
 $n = 1, 0$

$\frac{U}{16} + \frac{V}{4} > \frac{U}{8}$

$4V > U$

where charge order is more stable than uniform metal



Variable Range Hopping in a random system

N. F. Mott, *Phil. Mag.*, 19, 835 (1969).

Conductor $\sigma \sim 1/L \rightarrow$ Localization $\sigma \sim \exp(-\zeta r)$

$\sigma \propto \exp(-2\alpha R - W/k_B T)$ **R**: dopant distance
W: donor level depth

DOS \propto sphere volume of radius R
 $(4\pi R^3/3)D(E)dE \rightarrow W = 3/4\pi R^3 D(E)$

Itinerant \rightarrow E_F
Localized \rightarrow E_0

Mobility edge E_0

$\sigma \propto \exp(-2\alpha R - 3/4\pi R^3 D(E)k_B T)$

R maximizing $\sigma \rightarrow$ differentiate by R
 $2\alpha = 9/4\pi R^3 D(E)k_B T$

Localized limit at $R < R_0 \rightarrow -2\alpha R$ is important
 \rightarrow this R is inserted to give

$\sigma \propto \exp(-a/T^{1/4})$ **3D**
 $\sigma \propto \exp(-a/T^{1/3})$ **2D**
 $\sigma \propto \exp(-a/T^{1/2})$ **1D**

$\ln \rho$ 低温 $\ln \rho \propto 1/T^{1/4}$
高温 $\ln \rho \propto 1/T$

Anderson localization

Ordinary conductance $G = \sigma L^{d-2} \rightarrow$ **3D** $\rho = RS/L \Omega \text{cm}$
2D $\sim L^0$ sheet resistance

L x nu leads to $\frac{G(\nu L)}{G(L)} = f(G(L), \nu)$ **G(νL) and G(L) comes from ν**
(Scaling principle)

The logarithmic derivative is divided by ln ν, and in the ν → 1 limit
 $\lim_{\nu \rightarrow 1} \frac{\log(G(\nu L)/G(L))}{\log \nu} = \lim_{\nu \rightarrow 1} \frac{\log G(\nu L) - \log G(L)}{\log(\nu L) - \log L} = \frac{d \log G}{d \log L} \equiv \beta(G)$

Large G (metal) $\rightarrow \beta(G) = d - 2$ Power of L in G
Small G (localized) $\rightarrow G = G_0 e^{-\alpha L/\xi} \rightarrow \beta(G) = \log(G/G_0) = -\alpha L/\xi$

1D $\beta(G) < -1$ localization
2D $\beta(G) < 0$ weak localization
3D $\beta(G) = 0$ crossed
Anderson transition

(a)

(b)

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Percolation Theory

Black ball conducting
White ball insulating

Connected black cluster \rightarrow conducting

Critical black ball ratio coordination#

3D	Simple cubic	0.311	6
	Body center cubic	0.246	8
	Face center cubic	0.198	12
2D	Square lattice	0.593	4
	Triangular lattice	0.500	3
1D		~ 1.0	

Only a white ball makes insulator

Large coordination # leads to small critical values

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